# Chirality Conservation and Soft-Pion Production in Pion-Nucleon Collisions*†

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The formalism for soft-pion emission previously developed on the basis of the hypothesis of chirality conservation is applied to the production of soft pions in pion-nucleon collisions. Specific predictions are calculated for the class of least inelastic production events in which extraneous effects such as final-state interactions are most nearly under control. For these events angular distributions of the inelastically scattered projectile pions are calculated very simply from the corresponding phenomenological elastic-scattering data. Comparison of these results is made with the only relevant experiment, and no contradiction is indicated with the angular distributions, although the magnitudes of the cross sections seem to be significantly underestimated by a factor of about 7. The high-energy predictions of the formalism are summarily indicated. The relation of the chirality conservation formalism to pion production in peripheral processes suggests that these processes are to be understood in terms of symmetry-violating effects.

## I. INTRODUCTION

THE hypothesis of an approximately conserved-axial-vector current has been entertained speculatively by several authors.[1] As an analog to the conserved-vector current in the universal Fermi interaction proposed by Gell-Mann and Feynman,[2] an almost-conserved axial-vector current was suggested as a similar means of accounting for the meager renormalization exhibited in the Gamow-Teller part of the beta-decay interaction.[3] The conservation of an axial-vector current, or equivalently, invariance under $\gamma_5$ transformations, has also recently been accorded a fundamental significance by Nambu and Jona-Lasinio in a dynamical model for a theory of elementary particles based on an analogy with superconductivity.[4,5]

Within the context of a universal Fermi interaction the conserved-axial-vector hypothesis immediately lead to two disappointments, if the conservation were to be exact. First, it would prohibit pion decay.[6] Second, it would lead to an intolerably large induced pseudoscalar interaction in beta decay.[7] Accommodation of these two phenomena with a universal $V-A$ weak interaction then seemed to prescribe a degree of *non*conservation which was associated with the finite mass of the physical pion. The conjecture of such an almost conserved current has been used to illuminate the intriguing Goldberger-Treiman relation.[1] This relation among the coupling constants for pion decay, axial-vector beta decay, and pseudoscalar pion-nucleon coupling is well satisfied experimentally, and has previously represented about the only positive accomplishment attributable to the hypothesis of a conserved-axial-vector current.

$\gamma_5$ invariance is emphasized as an exact symmetry of the Lagrangian of a fundamental fermion field in the nonlinear superconductivity model of elementary particles. The physical solutions are not $\gamma_5$ eigenstates, but due to a physically unobservable degeneracy of the ground state under $\gamma_5$ transformations the invariance manifests itself physically as a *hidden* symmetry. A pseudoscalar meson field (pion) arises in that theory necessarily and automatically as bound states of fermion pairs to preserve $\gamma_5$ invariance when the bare mass of the fundamental fermion is nonzero.

We propose to examine some of the consequences of chirality conservation in pion-nucleon interactions. We do not need to presuppose any specific model Lagrangian. Of course, with the finite mass of the physical pion, it becomes implicit that this conservation is again only approximate; although we assume the pion-nucleon system to become chiral-invariant with the vanishing of the pion mass.

The axial-vector current that occurs in real phenomena involving nucleons and pions is a vector in isospace; e.g., the nucleon current in the Gamow-Teller $\beta$-decay interaction. The fourth component of this current is the density of the quantity we will call *chirality*. For example, the chirality of a free nucleon can be taken as $\bar{\psi}\gamma_4\gamma_5\boldsymbol{\tau}\psi$. The $\gamma_5$-gauge transformations associated with the conservation of such a current are of the type $\psi(x)\rightarrow\exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}\gamma_5)\psi(x)$, where $\boldsymbol{\alpha}$ is an arbitrary isovector.

The hypothesis of over-all conservation of chirality, by which changes in the nucleon chirality are compensated for in the pion chirality with pion emission, has been previously developed to yield a novel relation between the amplitudes for a given process and the associated process where an additional soft pion is also

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[1] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); J. Berstein, M. Gell-Mann, and L. Michel, *ibid.* **16**, 560 (1960); J. Berstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); F. Gürsey, Nuovo Cimento **16**, 230 (1960); F. Gürsey, Ann. Phys. (N. Y.) **12**, 91 (1961).

[2] S. S. Gershtein and J. B. Zeldovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [translation: Soviet Phys.—JETP **2**, 576 (1957)]; R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[3] S. Bludman, Nuovo Cimento **9**, 433 (1960); also see Ref. 1.

[4] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[5] G. Jona-Lasinio and Y. Nambu, Phys. Rev. **124**, 246 (1961).

[6] J. C. Taylor, Phys. Rev. **110**, 1216 (1958); M. L. Goldberger and S. B. Treiman, *ibid.* **110**, 1478 (1958).

[7] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

emitted.[8,9] Examples of such processes include soft-pion emission in inelastic collisions with nucleons by pions, nucleons, electrons, or neutrinos and by photoproduction. The latter three processes have been previously treated by means of an extension of the formalism on which the present work is based to allow inclusion of interactions which violate the chiral invariance, but are physically weak compared with the basic pion-nucleon interactions.[9] A generalized Kroll-Ruderman theorem was thus developed by which the axial-vector form factor could be determined from data on electroproduction or weak production of soft pions. Here, we wish to consider the predictions of the formalism for the pion-nucleon processes.

The formalism by which the conservation of chirality is responsible for soft-pion emission is briefly reviewed in the next section. Noncovariant calculations with this formalism are then applied to soft-pion production in pion-nucleon collisions in Sec. III A method of noncovariant calculation is employed by which emission of especially chosen classes of soft pions can be described by the corresponding elastic differential cross sections. Comparison of the calculated results with the very meager experimental results suggests substantial agreement in the general trends in angular distributions of the inelastically scattered primary pions; however, the cross sections that result seem to be too small in magnitude. In Sec. IV a covariant formulation of our method is examined as to its asymptotic behavior at very high energies, and simple asymptotic relationships between the cross sections for elastic scattering and the corresponding soft-pion emission process are derived. The relationship of the chirality-conservation formalism to the peripheral model for pion-production processes is considered. The possibility that $\gamma_5$ symmetry-violating effects are involved in these peripheral processes is also considered.

## II. FORMALISM

To establish our formalism we recapitulate its development as set forth in NL and NS. We postulate an axial-vector current, $\chi_\mu$, that is locally conserved according to $\partial_\mu\chi_\mu=0$. The fourth component of this current yields, in any given Lorentz frame, a constant of the motion, the total chirality, $\chi\equiv -i\int\chi_4 d^3x$.

The physical constant of the motion to which our conservation law refers is the expectation value of the chirality, since $\chi$ cannot be diagonal between physical eigenstates. The conservation of chirality in this manner derives from a physically unobservable degeneracy of the vacuum under the infinitesimal gauge transformations that are generated by the operator. These peculiarities of the conservation of chirality have been more thoroughly discussed in NL.

[8] Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962); hereafter referred to as NL.

[9] Y. Nambu and E. Shrauner, Phys. Rev. **128**, 862 (1962); hereafter referred to as NS.

As the distances between the particles become very large, $\chi$ is assumed to become asymptotically separable into a part, $\chi_N$, associated with the asymptotic nucleon field, and a part, $\chi_\pi$, associated with the asymptotic pion field. As suggested, for instance, by the general form of the axial-vector vertex function of the nucleon, or by the actual form of the axial current in the simple derivative coupling model of meson theory, we might expect the renormalized pion chirality to take asymptotically a form such as

$$\chi_\pi{}^{\text{in,out}}=-i\frac{2m}{g}\int\chi_{\pi,4}{}^{\text{in,out}}d^3x=\frac{2m}{g}\int\dot{\varphi}^{\text{in,out}}d^3x,\quad(2.1)$$

where $\varphi^{\text{in,out}}$ is the renormalized asymptotic pion field operator and $g/2m=f$ has been shown in NS to be the usual renormalized pion-nucleon coupling constant as defined in dispersion theory. This simple form may be looked upon as a phenomenological manifestation of the renormalized asymptotic chirality operator derived from a perhaps more complex operator form. There might in general be included in $\chi_\pi$ nonlinear terms; but we assume, as usual, that their nonlinear correlation effects vanish as the particles become asymptotically separated. The asymptotic operators $\chi_\pi{}^{\text{in,out}}$ will involve single-pion emission (or absorption) due to this simple asymptotic linearity in the pion field operator. Similarly, $\chi_N{}^{\text{in,out}}$ is assumed to become asymptotically independent of the pion field, and as a consequence, should contribute no effect that would alter the number of pions in the asymptotic states.

The essential basis of our formalism is the conservation of chirality as expressed in its commutation with the $S$ matrix,

$$\chi^{\text{out}}=S^{-1}\chi^{\text{in}}S=\chi^{\text{in}}.\quad(2.2)$$

Due to the assumption of the asymptotic separation of $\chi_N$ and $\chi_\pi$, we have,

$$S\chi_N{}^{\text{in}}-\chi_N{}^{\text{in}}S=\chi_\pi{}^{\text{in}}S-S\chi_\pi{}^{\text{in}}.\quad(2.3)$$

Taking matrix elements of this equation between asymptotic states of the parent (elastic) channel, the side involving $\chi_N{}^{\text{in}}$ simply gives

$$\langle f|[S,\chi_N{}^{\text{in}}]|i\rangle=i(2\pi)^4\delta^4(P_f-P_i)\langle f|[M,\chi_N{}^{\text{in}}]|i\rangle.\quad(2.4)$$

$M$ is the transition-amplitude operator for the elastic parent process, and $P_i$ and $P_f$ are 4-momenta of the initial and final states, respectively.

For the matrix element of $[\chi_\pi{}^{\text{in}},S]$, we use the asymptotic property described above by which $\chi_\pi{}^{\text{in}}$ has matrix elements connecting states of the parent process with intermediate states which differ from the original by a single meson, e.g., $\langle f,k|\chi_\pi{}^{\text{in}}|f\rangle$.

$$\langle f|[\chi_\pi{}^{\text{in}},S]|i\rangle=\sum_k\left(\langle f|\chi_\pi{}^{\text{in}}|f,k\rangle\langle f,k|S|i\rangle-\langle f|S|i,k\rangle\langle i,k|\chi_\pi{}^{\text{in}}|i\rangle\right).\quad(2.5)$$

Using the explicit form (2.1) above for $\chi_\pi{}^{\text{in}}$, its asymptotic matrix elements are simply

$$\langle i,k|\chi_\pi{}^{\text{in}}|i\rangle=-\langle f|\chi_\pi{}^{\text{in}}|f,k\rangle=(2m/g)ik_0(2k_0)^{-1/2}(2\pi)^3\delta^3(\mathbf{k}).\quad(2.6)$$

The off-diagonal $S$-matrix elements in Eq. (2.5) involve the radiative (absorption) transition amplitude, $M_r$, for the associated process in which an additional pion is emitted (absorbed). Consistently with the $S$-matrix notation that is obviously implicit in Eq. (2.4), we define $M_r$ according to

$$\langle f,k|S|i\rangle = i(2\pi)^4\delta^4(P_{f+k}-P_i)(2k_0)^{-1/2}M_r. \quad (2.7)$$

Using the crossing symmetry of $\langle f,k|S|i\rangle$, inserting these matrix elements into Eq. (2.5), and dividing out the energy-momentum factors, we obtain

$$i(2m/g)M_r{}^\alpha = \chi_N{}^\alpha M - M\chi_N{}^\alpha. \quad (2.8)$$

This is the central relation of our formalism. $M_r{}^\alpha$ is the amplitude for the associated emission of an additional pion of vanishing energy. $\alpha$ is the isospin index corresponding to the state in which the pion is emitted. This fact that chirality conservation strictly applies only to pions of vanishing energy is, of course, the principal difficulty for phenomenological applications of our relation (2.8).

$\chi_N{}^\alpha$ in Eq. (2.8) can be specified so that the chirality operator of a free nucleon is $\chi_N{}^\alpha = \tau^\alpha\gamma_4\gamma_5$. This normalization is consistent with the assumptions made in writing Eq. (2.1) for the pion part of the asymptotic chirality, $\chi_\pi{}^\alpha$. In the remainder of this section we will, for simplicity, consider our process as involving only a single nucleon.

To apply our relation (2.8) directly to the amplitude $M$, as it ordinarily occurs, the positive-energy-projection operators for the nucleon must be explicitly inserted, yielding the form

$$i\frac{2m}{g}M_r{}^\alpha = \tau^\alpha\left(\frac{m}{E'}\gamma_4\gamma_5-\gamma_5\right)M + M\left(\frac{m}{E}\gamma_5\gamma_4-\gamma_5\right)\tau^\alpha \quad (2.9)$$

which, when applied to positive-energy Dirac spinors, is equivalent to

$$i(2m/g)M_r{}^\alpha = -\tau^\alpha\boldsymbol{\sigma}\cdot\mathbf{v}'M + M\boldsymbol{\sigma}\cdot\mathbf{v}\tau^\alpha. \quad (2.10)$$

Here, $\mathbf{v}$ is the velocity operator for the nucleon, $\mathbf{v}=\mathbf{p}/E$. The form (2.10) reveals some of the physical nature both of the operator $\chi_N{}^\alpha$ and of our soft-pion radiation mechanism as a whole. In fact, we might have more intuitively, but equivalently, defined the nucleon chirality in terms of helicity, $h$, as $\chi_N{}^\alpha = -\tau^\alpha h|\mathbf{v}| = -\tau^\alpha\boldsymbol{\sigma}\cdot\mathbf{v}$.

To facilitate further development of our formula as well as further description of the process to which it applies, we can rearrange Eq. (2.9) into the form,

$$i(2m/g)M_r{}^\alpha = (m/E)[\tau^\alpha\gamma_4\gamma_5,M] - \{\tau^\alpha\gamma_5,M\}. \quad (2.11)$$

The mathematical origins and likewise the physical interpretations of the two types of terms which appear in this form are, although similar, nevertheless quite distinct.

The $\gamma_5$ anticommutator is characteristic of the infinitesimal $\gamma_5$ gauge transformation which generates the change in $M$ that is to be compensated in the pion field leading to $M_r{}^\alpha$. This term represents here the effects of the infinitesimal transformation on the internal structure of the amplitude $M$. In the sense that a generalized Ward identity[10] can be associated with the infinitesimal $\gamma_5$ transformation, this term is the sum of the changes in $M$ such that successively in each propagator corresponding to an internal nucleon line a $\gamma_5$ vertex is inserted and an external soft-pion line affixed. Thus, this $\gamma_5$ anticommutator represents in Eq. (2.11) the contribution of *internal* pion bremsstrahlung, or structure radiation to $M_r{}^\alpha$.

The commutator in Eq. (2.11) is similarly the contribution from the changes generated by the $\gamma_5$ transformation in the propagators corresponding to *external* nucleon lines. Each term of the commutator is the limit of a nucleon pole term in which the pion momentum vanishes followed by the vanishing of the pion mass.[11] This limiting process can be formally expressed by

$$\frac{i}{g}M_r{}^\alpha = -\tau^\alpha i\gamma_5\frac{i}{i\gamma\cdot(p'+k)+m}M - M\frac{i}{i\gamma\cdot(p-k)+m}i\gamma_5\tau^\alpha - \{\tau^\alpha\gamma_5,M\}, \quad (2.12)$$

$$\xrightarrow[\mu\to 0]{\lim}\left[\xrightarrow[|\mathbf{k}|\to 0]{\lim}\left(\frac{1}{2E}[\tau^\alpha\gamma_4\gamma_5,M] - \frac{1}{2m}\{\tau^\alpha\gamma_5,M\}\right)\right]. \quad (2.13)$$

$k$, $p$, $p'$ in these equations are, respectively, 4-momenta of the soft pion and the in and out states of the nucleon from which it is emitted. These pole contributions are represented by diagrams (a) and (b) of Fig. 1, and the

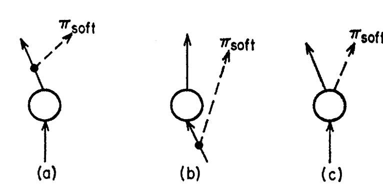


FIG. 1. Diagrams (a) and (b) represent the nucleon-pole terms generated by the infinitesimal $\gamma_5$ transformation as changes in the propagators of the external nucleon lines, and (c) represents the internal bremsstrahlung corresponding to the $\gamma_5$ anticommutator in Eq. (2.11).

[10] J. C. Ward, Phys. Rev. **77**, 293 (1950); Y. Takahashi, Nuovo Cimento **6**, 371 (1957); E. Kazes, *ibid.* **13**, 1226 (1959); J. Bernstein, M. Gell-Mann, and L. Michel, *ibid.* **16**, 560 (1960).

[11] For a straightforward derivation of these results which emphasizes the similar origins of the two types of radiative terms, we can simply include the propagators for the external nucleon lines with $M$ at the outset and then proceed to calculate directly the change in the improper amplitude $S_FMS_F$ resulting from the infinitesimal $\gamma_5$ transformation $\psi\to\exp(i\gamma_5\delta\boldsymbol{\varphi}\cdot\boldsymbol{\tau})\psi$.

$$\begin{aligned}\delta(S_FMS_F) &= i(2m/g)S_FM_r{}^\alpha\delta\varphi^\alpha S_F = \{i\gamma_5\delta\boldsymbol{\varphi}\cdot\boldsymbol{\tau}, S_FMS_F\} \\ &= 2mS_F(-i\gamma_5\boldsymbol{\tau}\cdot\delta\boldsymbol{\varphi}S_FM - MS_F\boldsymbol{\tau}\cdot\delta\boldsymbol{\varphi}i\gamma_5 \\ &\qquad -\{\gamma_5\boldsymbol{\tau}\cdot\delta\boldsymbol{\varphi}/2m, M\})S_F. \quad (2.13')\end{aligned}$$

For calculations in lowest order, $\delta\boldsymbol{\varphi}$ is to be treated here as a classical-external test field; and then identified with the "soft" pion, which, of course, we must in turn hope also to identify with the real pion produced in physical reactions.

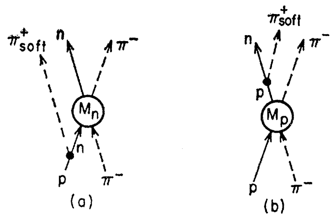


FIG. 2. The nucleon pole represented in (a) contributes to the term $\boldsymbol{\sigma}\cdot\mathbf{v}'M_p$ in Eq. (2.15), while the pole represented in (b) contributes to $M_n\boldsymbol{\sigma}\cdot\mathbf{v}$, each in a one-to-one correspondence. There remain, of course, contributions to each of these terms which are not associated with the nucleon poles; the internal bremsstrahlung contributions from the respective halves of the $\gamma_5$ anticommutator, as given in Eq. (2.11).

internal bremsstrahlung contributions of the $\gamma_5$ anticommutator are included there in diagram (c).

With the identification of the pole terms, the resemblance of our soft-pion emission amplitude to the case of soft-photon bremsstrahlung, as treated by Low,[12] becomes apparent. He showed that terms of order $k^{-1}$ and unity in the photon momentum of the bremsstrahlung amplitude are given by the same diagrams as we have ascribed above to our amplitude (2.12). The pion bremsstrahlung problem has no infrared divergence, but, as previously discussed in NL and NS, exhibits the singularity of the nucleon propagator as an ambiguity in the limiting procedure, depending on the direction from which the pion momentum approaches zero in space-time. The limit $|\mathbf{k}|\to 0$ should precede the limit $\mu\to 0$.

The expression (2.12) is explicitly covariant and, furthermore, it agrees with the exactly correct expression for the radiative amplitude for emission of pions of arbitrary momenta as far as the pole terms are concerned. These pole terms are well defined in the sense of dispersion theory, and so they specify our coupling constant completely. Their identification and separation can be important for the description of soft-pion emission at high energy, since one or the other type of term may become exclusively dominant. For example, in the $\pi N\to\pi N\pi_{soft}$ process these nucleon-pole terms are prohibited by parity conservation from contribution to production in a peripheral collision where a single virtual pion is exchanged. Furthermore, if at very high energies an asymptotic $\gamma_5$ invariance prevails independently of the pion field, then the $\gamma_5$ anticommutator term in our amplitude will also vanish. Discussion along these lines will be presented later in Sec. IV.

The causal development of the process described by our formula can be illuminated by observing the charge operations it describes. Consider, for definiteness, the specific example of the process $\pi^-p\to\pi^-n\pi_{soft}{}^+$. Our formula for the radiative amplitude has the explicit form,

$$i(2m/g)\langle n|M_r|p\rangle=\langle n|-\tau^-\boldsymbol{\sigma}\cdot\mathbf{v}'M+M\boldsymbol{\sigma}\cdot\mathbf{v}\tau^-|p\rangle. \quad (2.14)$$

[12] F. E. Low, Phys. Rev. **110**, 978 (1958).

The preoperative charge operator, $\tau^-$, can be removed from this equation by taking explicitly into account its operation on the nucleon isospinor to the right, giving $\tau^-|p\rangle=\sqrt{2}|n\rangle$. The remaining amplitude $\langle n|M|n\rangle$ is just the isospin projection appropriate to elastic scattering in the $\pi^- - n$ charge state, $M_n$. Thus, $\tau^-$ in $M\chi_N$ changes the incoming proton into a neutron emitting the soft $\pi^+$, after which this neutron proceeds to scatter internally, or virtually, with the incoming $\pi^-$. In the same manner the isospin significance of the postoperative nucleon chirality in the term $\chi_N M$ corresponds to virtual scattering in the incoming $\pi^-p$ system *followed* by $p\to n+\pi_{soft}{}^+$. We can formally summarize this example by writing the isospinor equation for the $\pi^-p\to\pi^-n\pi_{soft}{}^+$ radiative amplitude.

$$-i(2m/g)\langle n|M|p\rangle=\langle n|\tau^-\boldsymbol{\sigma}\cdot\mathbf{v}'M-M\boldsymbol{\sigma}\cdot\mathbf{v}\tau^-|p\rangle$$
$$=\sqrt{2}(\boldsymbol{\sigma}\cdot\mathbf{v}'M_p-M_n\boldsymbol{\sigma}\cdot\mathbf{v}). \quad (2.15)$$

Figure 2 illustrates the processes represented here. This interpretation of the causal development of the process represented in our formula is particularly suitable, since the $M$ occurring there is a mass-shell amplitude appropriate to real pion-nucleon scattering. However, this description of the causal development is rather schematic, since it actually pertains only to the pole terms and, although $\boldsymbol{\sigma}\cdot\mathbf{v}$ is definitely related to such a pole, it also includes a contribution from the nonpole part which we have called the internal bremsstrahlung term.

## III. $\pi N\to\pi N\pi_{soft}$ NONCOVARIANT

In this section we will proceed to apply the general formalism developed in the preceding section to the specific process of soft-pion emission in pion-nucleon collisions. We will attempt to develop specific results that may afford direct comparison with experiment, although the class of phenomena accessible to simple treatments may be quite restricted.

We will specify our amplitudes $M$ and $M_r$ for the elastic pion-nucleon scattering and the associated pion bremsstrahlung process according to the convention by which the pertinent $S$-matrix elements are

$$\langle p'q|S-1|pq\rangle=i(2\pi)^4\delta^4(p'+q'-p-q)\times\frac{\bar{u}_{p'}M(p'q';pq)u_p}{(4\omega'\omega EE'/m^2)^{1/2}}, \quad (3.1)$$

$$\langle p'q'k|S|pq\rangle=i(2\pi)^4\delta^4(p'+q'+k-p-q)\times\frac{\bar{u}_{p'}M_r{}^{\alpha}(p'q'k;pq)u_p}{(8\omega'\omega k_0EE'/m^2)^{1/2}}; \quad (3.2)$$

$p=(\mathbf{p},E)$ and $q=(\mathbf{q},\omega)$ are in each case the 4-momenta of the parent nucleon and pion, respectively.[13] $q'$ and $p'$ are the corresponding outgoing momenta of these particles. $k=(\mathbf{k},k_0)$ and $\alpha$ refer, respectively, to the 4-

[13] Our metric is specified by the product of two arbitrary 4-vectors $A$ and $B$: $A\cdot B=\mathbf{A}\cdot\mathbf{B}-A_0B_0=\mathbf{A}\cdot\mathbf{B}+A_4B_4$; $A_4=iA_0$.

momentum and charge index of the soft pion emitted.

The cross section formulas corresponding to the $S$-matrix elements (3.1) and (3.2) are, respectively,

$$d\sigma/d\Omega_{q'}=|M|^2{}_{\mathrm{av}}m^2/(4\pi W)^2\,,\quad\text{in the center-of-mass system;}\qquad(3.3)$$

and,

$$\frac{d^2\sigma}{d\omega' d\Omega_{q'}}=\frac{1}{F}|M_r|^2{}_{\mathrm{av}}\frac{m^2|\mathbf{q}'|\,|\mathbf{k}_Q|}{4(2\pi)^4\omega EQ}=\frac{|M_r|^2{}_{\mathrm{av}}m^2|\mathbf{q}'_C|\,|\mathbf{k}_Q|}{4(2\pi)^4U|\mathbf{q}_C|Q},$$
$$\text{in the center-of-mass system.}\qquad(3.4)$$

In these equations $F=[-(\mathbf{p}\omega-\mathbf{q}E)^2-(\mathbf{p}\times\mathbf{q})^2]^{1/2}/E\omega=[-\frac{1}{4}(p_\mu q_\nu-p_\nu q_\mu)^2]^{1/2}/E\omega$ is a generalized flux factor that reduces to the relative velocity of the two incoming particles in any Lorentz frame that moves colinearly with the relative motion. In both cases $F$ is the same for the common initial state. Also, once and for all, we define the invariants: $U=[-(p+q)^2]^{1/2}$ is the over-all center-of-mass (c.m.) energy of the incoming system, $W=[-(p'+q')^2]^{1/2}$ is the total energy of the two outgoing parent particles in their common c.m. system, $Q=[-(p'+k)^2]^{1/2}$ is the total energy of the outgoing nucleon and soft pion in their c.m. system. In the case of elastic scattering, or in the limiting case of zero-energy soft-pion emission, $W$ and $U$ would be equal. $\mathbf{k}_Q$ is the 3-momentum of the soft pion in the c.m. system of it and the outgoing nucleon where $\mathbf{p}'_Q+\mathbf{k}_Q=0$. Throughout we will designate 3-momenta and energies with subscripts $C$, $L$, $Q$, or $W$ accordingly as they are measured in the Lorentz frames where, respectively, $\mathbf{P}_C+\mathbf{q}_C=0$, $\mathbf{P}_L=0$, $\mathbf{p}'_Q+\mathbf{k}_Q=0$, or $\mathbf{p}'_W+\mathbf{q}'_W=0$.

$|M|^2{}_{\mathrm{av}}$ and $|M_r{}^\alpha|^2{}_{\mathrm{av}}$ in the above two cross section formulas are the usual averages over initial, and sums over final spin states of the squares of these two amplitudes. We have demonstrated a simple relation by which the radiative amplitude for soft-pion emission, $M_r$, is determined by the amplitude for the associated elastic parent process, $M$. Furthermore, we now note that in special cases for pion emission with vanishing inelasticity in particularly chosen Lorentz frames, $|M_r{}^\alpha|^2{}_{\mathrm{av}}$ depends only on the corresponding spin-averaged square, $|M|^2{}_{\mathrm{av}}$, and not on the actual detail of the parent amplitude, $M$. The noncovariant form of Eq. (2.10) for the radiative amplitude suggests the particular convenience of considering the emission of soft pions in those Lorentz frames where one of the nucleon velocities vanishes, viz., soft-pion ($s$-wave) emission in the laboratory, or emission of the soft pion in intimate accompaniment with the outgoing nucleon. In either of these special cases the vanishing of the chirality of a free nucleon with the vanishing of its velocity in a given frame results in a reduction of Eq. (2.10) to a single term by which $|M_r{}^\alpha|^2{}_{\mathrm{av}}$ separates into the simple form,

$$\begin{aligned}|M_r{}^\alpha|^2{}_{\mathrm{av}}&\equiv\tfrac{1}{2}\,\mathrm{Tr}M_r{}^{\alpha\dagger}M_r{}^\alpha\\&=(g/2m)^2\tfrac{1}{2}\,\mathrm{Tr}[(\tau^\alpha\boldsymbol{\sigma}\cdot\mathbf{v}M)^\dagger\tau^\alpha\boldsymbol{\sigma}\cdot\mathbf{v}M]\\&=(g/2m)^2\tfrac{1}{2}\mathbf{v}^2\langle\tau^\alpha\rangle^2\,\mathrm{Tr}M^\dagger M\\&=(g/2m)^2\langle\Delta\chi_N\rangle^2|M|^2{}_{\mathrm{av}}\,,\end{aligned}\qquad(3.5)$$

where $\langle\tau^\alpha\rangle^2$ takes the value 2 for emission of charged pions and unity for emission of neutral pions; $\langle p|\chi_N|p\rangle^2=\langle\Delta\chi_N\rangle^2=\langle\tau^\alpha\rangle^2(\boldsymbol{\sigma}\cdot\mathbf{v})^2$ is the change of nucleonic chirality squared; since $\langle p'|\chi_N|p'\rangle=0$, or vice versa, in the special cases we are considering. $M$ occurring in our formula for $M_r{}^\alpha$ is the mass-shell amplitude for real-scattering. Thus, in our cross-section formula, not only is $|M_r{}^\alpha|^2{}_{\mathrm{av}}$ just a function of $|M|^2{}_{\mathrm{av}}$, but moreover, this spin average of the square of the invariant amplitude $M$ is just the elastic cross section for real scattering given by Eq. (3.3). The separable form in which we have been able to cast $|M_r{}^\alpha|^2{}_{\mathrm{av}}$ allows the differential cross section for the inelastically scattered parent pion producing a pion at rest relative to the outgoing nucleon to be formulated simply as,

$$\frac{d^2\sigma}{d\omega'_C d\Omega_{q'_C}}=\left(\frac{g}{2m}\right)^2\langle\tau^\alpha\rangle^2\mathbf{v}_Q{}^2\frac{d\sigma(W,z_W)}{d\Omega_{q'_W}}\frac{W^2|\mathbf{k}_Q|\,|\mathbf{q}'_C|}{(2\pi)^2UQ|\mathbf{q}_C|},\qquad(3.6)$$

where $\mathbf{v}_Q$ is the velocity of the incoming nucleon in the rest frame of the outgoing pion-nucleon system.

Similarly, the cross section for the inelastically scattered parent pion producing a soft pion in the laboratory, i.e., at rest relative to the incoming nucleon, is

$$\frac{d^2\sigma}{d\omega'_C d\Omega_{q'_C}}=\left(\frac{g}{2m}\right)^2\langle\tau^\alpha\rangle^2\mathbf{v}_L{}^2\frac{d\sigma(U,z_C)}{d\Omega_{q'_C}}\frac{U|\mathbf{k}_Q|\,|\mathbf{q}'_C|}{(2\pi)^2Q|\mathbf{q}_C|}.\qquad(3.7)$$

$\mathbf{v}_L$ here is the nucleon recoil velocity in the laboratory frame.

According to our previous discussions, emission of the soft pion at rest relative to the outgoing nucleon precedes the elastic parent interaction, while emission at rest relative to the incoming nucleon follows the parent interaction. For this reason, $d\sigma/d\Omega_{q'}$ in Eqs. (3.6) and (3.7) is evaluated at $W$ and $z_W\equiv\cos\vartheta_W$ in the first case and at $U$ and $z_C\equiv\cos\vartheta_C$ in the second case. The actual calculations with these equations were made with external momenta and energies taking the physical values occurring in the real processes. The parent interaction vertex was considered at the center-of-mass energy of its two *external* incoming or outgoing particles and at the physical scattering angle of the parent pion in that frame. Of course, for true quasielastic emission of zero-energy pions $U^2$ would be equal to $W^2$, but the distinction has been maintained for purposes of accounting somewhat for the finite energies of the pions emitted in real processes; especially when we attempt to apply these formulas, perhaps unwarrantedly, at low energies where the pion mass is not small.

For calculations of radiative cross sections for emission of soft pions at rest relative to the outgoing nucleon with Eq. (3.6), the primary data were the experimental values of the differential cross section of the elastic parent process ($\pi N \to \pi N$) fed in directly in the form $d\sigma/d\Omega_{\mathrm{c.m.}} = \sum_{n=0}^{5} A_n z_{\mathrm{c.m.}}{}^n$, appropriate to its proper center-of-mass energy, $W$, and scattering cosine, $z_W$. $W$ and $z_W$ were determined from the external momenta of the parent particles as they participated in the kinematics of the production of a real pion in accompaniment of the outgoing nucleon: $W \equiv [-(p'+q')^2]^{1/2}$ and $z_W \equiv \mathbf{q}_W \cdot \mathbf{q}'_W / |\mathbf{q}_W| \, |\mathbf{q}'_W|$. A great simplification of kinematics at little sacrifice of principle, in view of restrictions already imposed, was gained by averaging over the $S$-wave distribution of the soft pion relative to the outgoing nucleon. The center-of-mass energy of this system, $Q \equiv [-(p'+k)^2]^{1/2}$, was parametrized as $Q = m + \mu + \epsilon$ and calculations performed for values of the parameter $\epsilon$ from 10 to 100 MeV. All other kinematic quantities in Eq. (3.6) were obtained by successive Lorentz transformations, first to the over-all center-of-mass system, thence to the laboratory frame, starting from given $W$ and $z_W$ in the proper frame of the parent process and $d\sigma(W,z_W)/d\Omega_{q'_W}$.

Calculations with Eq. (3.7) for emission of soft pions in the laboratory frame were carried out in a similar manner starting with $U \equiv [-(p+q)^2]^{1/2}$ and $z_C \equiv \mathbf{q}_C \cdot \mathbf{q}'_C / |\mathbf{q}_C| \, |\mathbf{q}'_C|$, the total energy and scattering angle in the over-all center-of-mass system. The latter is the proper frame of the parent interaction; since in this case, as mentioned above, the soft-pion emission is associated only with the outgoing nucleon current. An angular average over the $S$-wave pion-nucleon system was again invoked, setting $k_{0L} = \mu + \epsilon$, with $\epsilon$ taking values from 10 to 50 MeV. Then, with energy conservation and specified $U$ and $z_L$ all kinematic quantities in formula (3.7) were determined by Lorentz transformation to the over-all center-of-mass system.[14]

The kinematic analyses described here for our treatment are quite similar, both in form and spirit, to those employed for phenomenological applications of the so-called Chew-Low[15] approximation in the peripheral model or of the isobar model[16] in the sense that all have in common that internal interaction amplitudes are determined from their mass-shell values by one rationalization or another, and then data from physical processes can be fed into them on this basis, directly, or with various degrees of sophistication and accommodation.

The results of applying Eqs. (3.6) and (3.7) to elastic pion-proton data covering a wide range of energies are shown in Figs. 3–6.[17] The ordinate of each cross-section curve should be multiplied by a factor of 2 when the produced soft pion is charged; corresponding to the factor $\langle \tau^\alpha \rangle^2$ in Eqs. (3.6) and (3.7). Obviously, other than minor kinematic effects of momentum and energy conservation, the distributions are just the appropriate elastic-differential cross sections multiplied by the square of the change of nucleon chirality, $\langle \tau^\alpha \rangle^2 v^2$. This factor emphasizes tremendously any nonzero scattering in the backward hemisphere and strongly reduces forward scattering, which would even vanish at $\vartheta = 0$ if the pion mass were ignored kinematically. For all the curves shown $\epsilon$, the $Q$-value parameter described above, has the value 10 MeV. At higher values of the $Q$-value parameter the effects of final-state interactions might tend to obscure the relevance of our results to real pion phenomena.

The results calculated with Eq. (3.6) and with Eq.

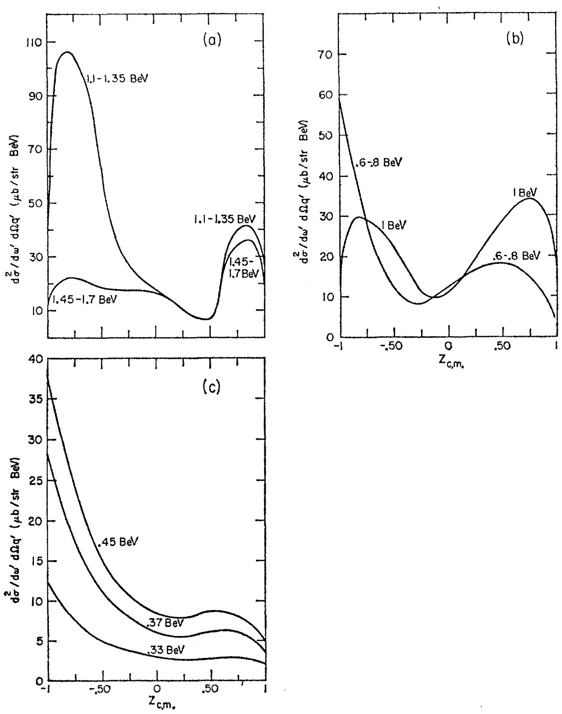


FIG. 3. Calculated angular distributions of the inelastically scattered projectile pion in the center-of-mass system for the processes $\pi^- p \to \pi^- p \pi_{\mathrm{soft}}{}^0$ and $\pi^+ p \to \pi^+ n \pi_{\mathrm{soft}}{}^-$ when the soft pion is emitted in accompaniment with the outgoing nucleon. The curves are labeled by the incident kinetic energies in the laboratory system. When the soft pion emitted is charged, the ordinate scale should be multiplied by 2. These calculations are based directly on elastic $\pi^- p$ scattering data as given in Ref. 17.

[14] All of the numerical calculations were actually performed on the IBM-1620 computor at the Enrico Fermi Institute for Nuclear Studies.

[15] G. F. Chew, Phys. Rev. **112**, 1380 (1958), G. F. Chew and F. E. Low, *ibid.* **113**, 1640 (1959).

[16] S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **105**, 1874 (1957); **106**, 1107 (1957); **109**, 1723 (1958); Phys. Rev. Letters **5**, 24 (1960).

[17] The $\pi^- p$ elastic scattering data used to calculate the angular distributions shown in Figs. 3 and 4 came from the following published sources: C. D. Wood, T. J. Devlin, J. A. Helland, M. J. Longo, B. J. Mayer, and V. Perez-Mendez, Phys. Rev. Letters **9**, 481 (1961); J. Deahl, M. Derrick, J. Fetkovich, T. Fields and G. B. Yodh, in *Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 187; L. Goodwin UCRL-9119, 1960 (unpublished); Y. Budagov, S. Wiktor, V. P. Dzhelepov, P. F. Yermolov, and V. I. Moskalev, in *Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 190. The $\pi^+ p$ elastic scattering data used to calculate the angular distributions shown in Fig. 5 came from U. Bidan and F. Levy, Nuovo Cimento **24**, 334 (1962).

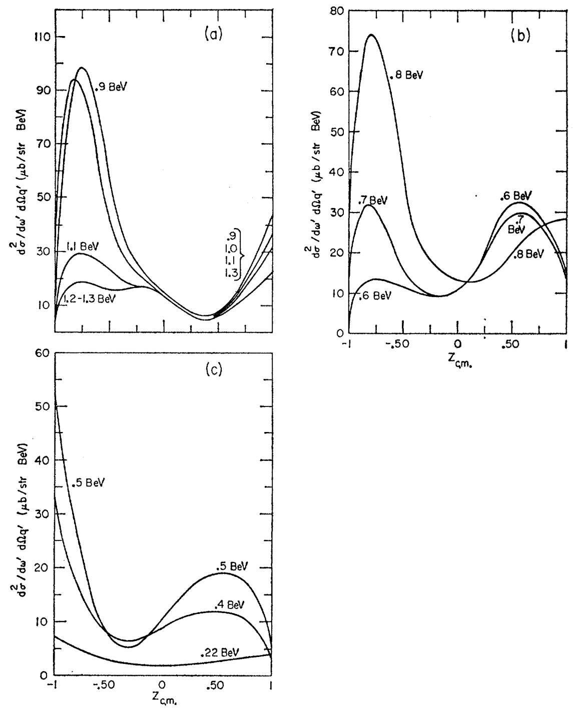


FIG. 4. Angular distributions for the same processes as are described for Fig. 3 except that here the emitted pions are "soft" in the laboratory frame instead of in the rest frame of outgoing nucleon.

(3.7) are quite similar, as may be seen in Figs. 3–5. Both classes of events represented by these two formulas can be readily selected experimentally; but for directness, since at present there is insufficient experimental data for quantitative comparisons, we will confine our attention to the first case only .The selection of events where the soft pion comes off close to the outgoing nucleon can be quite directly accomplished by observing the inelastically scattered parent pions having nearly maximal momentum in the center of mass. Most importantly, however, final-state interactions, which would, in general, be incomprehensible in our treatment, are the least so for this class of events; since we specify the final state of the soft pion to have zero (kinetic) energy and angular momentum relative to the outgoing nucleon, and the virtual elastic scattering of the two parent particles already includes phenomenologically the mass-shell part of their final-state interaction. It should be remembered that in our formalism these events proceed via the radiation of the change of nucleonic chirality in the form of the soft pion *followed* by virtual scattering of the parent particles in the final state. So, neglecting pion-pion interactions, the pion-nucleon final-state interactions that are ignored in our treatment of this class of "least inelastic" events should be of no greater order than the internal off-mass-shell effects otherwise inherently ignored in the application of our formalism to real pion phenomena.

In Fig. 6 our calculated angular distributions are compared with the available data for pion production by pions of 1.3-BeV kinetic energy in the laboratory.[18] These seem to be the only published experimental results available at the present from which the angular distributions of the minimally inelastic events accessible to our treatment can be obtained. The data for comparison were selected, in accordance with the prescription described above, to include only those experimental events for which the inelastically scattered fast pions retained momenta within some specified tolerance range of the maximum momentum. This then insured that the internal energy of the relative motion in the system of the outgoing nucleon and soft pion produced not be large. To glean any appreciable statistics this point had to be stretched somewhat, although we can argue that this may not be serious.

The justifying reason that it may not be crucial as to where the line is drawn in this compromise is that phenomenologically the angular distribution of the inelastically scattered fast pions appears to be rather independent of their energy. Thus, the sum over the whole energy spectrum of these pions also yields an angular distribution which shows a very strong similarity to our calculated angular distribution for the least inelastic events.[18] In fact, throughout the range of incident pion energies represented in Figs. 3 to 5 we have found quite striking thematic similarities between the calculated angular distributions shown there and experimental distributions of the totality of pions scattered with all possible energies. Another reason that the momentum selection of our comparison sample may not be overbearingly crucial is that the angular distribution which we calculate for the inelastically scattered pions is not strongly dependent on the region of phase space we choose in which to identify the pions we call soft. This may be seen in the similarity between the curves in Figs. 3 and 5(a) for events where the soft pions come

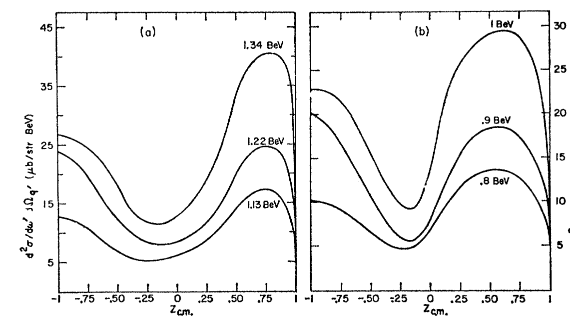


FIG. 5. Calculated angular distributions of the inelastically scattered projectile pion in the center-of-mass system for the processes $\pi^+p \rightarrow \pi^+p\pi_{\rm soft}{}^0$ and $\pi^-p \rightarrow \pi^-n\pi_{\rm soft}{}^+$. (a) shows the distributions for when the soft pion is emitted in accompaniment of the outgoing nucleon, and (b) shows the distribution for emission of soft pions in the laboratory frame. The number labeling each curve refers to the incident kinetic energy in the laboratory. For emission of charged soft pions the ordinate scales must be read larger by a factor of 2. These calculations are based directly on elastic $\pi^+p$ scattering data as given in Ref. 17.

[18] W. D. Shephard and W. D. Walker, Phys. Rev. **126**, 278 (1962). The experimental data for comparison in Figs. 6 and 7 above were taken from the scatter diagrams in Figs. 3 and 4, respectively, of this reference.

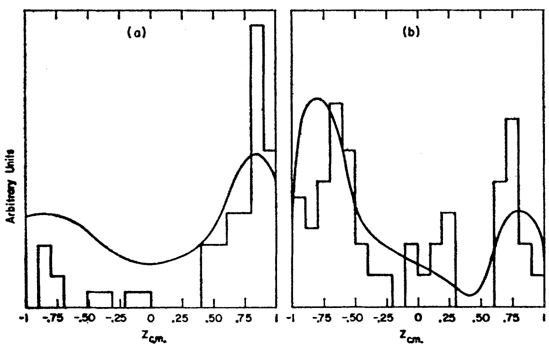


FIG. 6. Comparison of our calculated angular distributions with the experimental data of Shephard and Walker (Ref. 18) for the least inelastic events where the soft pion is produced in accompaniment with the outgoing nucleon. Figure 6(a) refers to the channel $\pi^- p \to \pi^- n \pi_{\text{soft}}^+$, and (b) refers to $\pi^- p \to \pi^- p \pi_{\text{soft}}^0$. The histograms shown here include events in which the inelastically scattered pions retained momenta within, (a), 55 MeV/$c$ of the maximum in the charged production channel, and (b), within 65 MeV/$c$ of the maximum momentum in the neutral-production channel.

off with the outgoing nucleon and the corresponding curves in Figs. 4 and 5(b) for the production of soft pions in the rest frame of the incoming target particle, i.e., the lab.

Some estimate of the predicted magnitudes of the pion-production cross sections can be made by integrating over angle the distributions given above, then performing the energy integration by assuming a linear dependence on $|\mathbf{k}_Q|$ in Eq. (3.4) over the narrow strip along the border of phase space that corresponds to our least inelastic events. Making such an approximation, we get for the cross section over the strip of phase space to which the differential cross section (3.6) applies,

$$\sigma_\epsilon = \left[ \int \frac{d^2\sigma}{d\omega' d\Omega_{q'}}\bigg|_{\epsilon_0} d\Omega_{q'} \right] \frac{(2\mu)^{1/2}\epsilon^{3/2}}{3|\mathbf{k}_Q(\epsilon_0)|}, \tag{3.8}$$

where $\mu$ is the pion mass and $\epsilon$ is the $Q$-value parameter described earlier, whose magnitude is approximately related to the strip width by $d\omega' = -(Q/U)d\epsilon$. $\epsilon_0$ is a fiducial value of $\epsilon$.

For $\pi^- p \to \pi^- n \pi_{\text{soft}}^+$, the angular integral of the distribution shown in Fig. 6 gives about $\int (d^2\sigma/d\Omega_{q'} d\omega') d\Omega_{q'} \simeq 1.1$ mb/BeV. So a strip width corresponding to $\epsilon \lesssim 80$ MeV in Eq. (3.8) gives $\sigma_\epsilon \simeq 83$ $\mu$b. Experimentally the total cross section is $\sigma(\pi^- p \to \pi^- n \pi^+) = 9.2$ mb corresponding to 259 events. Of these, 18 events are contributed from our strip[18]; i.e., (18/259)9.2 mb$\simeq$0.6 mb is the experimental value to which $\sigma_\epsilon$ pertains. Similarly, our calculation in the $\pi^- p \to \pi^- p \pi^0$ channel give $\sigma_\epsilon \simeq 42$ $\mu$b, to be compared with about 0.32 mb contributed from the strip experimentally. The fact that our calculation underestimates the cross sections in the strip by about the same factor of 7 in both channels may indicate that the calculation is not chiefly in error due to overlap of the $p$-wave resonance into our strip. If such were the case, the neutral production might be expected to show twice as large a discrepancy ratio as the production of charged pions.

The noticeable underestimation of the cross sections is rather serious, as we have already indicated that it is unlikely that final-state interactions in this configuration should alone account for the full discrepancy. The discrepancy may not be quite so great, as we have tried to be quite generous in our estimates, since they are based on such niggardly data. There seems to be no conclusive argument from the point of view from which our formalism is developed that indicates whether one should expect over- or underestimation of the soft-pion-production cross sections. It may be that symmetry-violating effects contribute strongly toward making up this discrepancy. Peripheral interactions are supposed to be significant in pion production in this energy region, and these contributions will be discussed in Sec. IV as possible results of symmetry-violating effects.

## IV. COVARIANT TREATMENT AND HIGH-ENERGY BEHAVIOR OF SOFT-PION EMISSION

The covariant analysis described in Sec. II, by which the radiative amplitude can be decomposed into contributions from what we have called internal and external emission, allows us to consider each of those two modes of emission separately as possibly dominating at high energies. Considering in this manner the possible contributions from peripheral interactions, we will point out that those processes are likely to be associated with $\gamma_5$ symmetry-violating effects that may be more important than the chirality-conserving effects previously considered.

The usual form of the covariant spinor amplitude for elastic pion-nucleon scattering is

$$M = -a + i\tfrac{1}{2}\gamma \cdot (q+q')b, \tag{4.1}$$

where, as before, $q$ and $q'$ are the initial and final 4-momenta, respectively, of the pion. The amplitudes $a$ and $b$ are functions of the invariants $s = -(p+q)^2$ and $t = -(p-p')^2$, where $p$ and $p'$ are, respectively, the initial and final 4-momenta of the nucleon.

If at high energies a semiclassical limit exists where spin and isospin effects become unimportant and elastic scattering is described by a single amplitude, then we can apply these conditions and ascertain the corresponding mathematical relations for the spinor amplitudes. Thus, the vanishing importance of the spin-flip amplitude in the classical limit requires that for scattering in a given charge state $\lim_{s\to\infty} a = s^{1/2} b$, etc. Similarly, the vanishing importance of isospin flip; i.e., of charge-exchange scattering, requires that $\lim_{s\to\infty} a_p = a_n$, etc., where $a_p$ and $a_n$ refer to elastic scattering in the different charge states. With these conditions the elastic cross section for pion-nucleon scattering in the high-energy semiclassical limit becomes

$$\frac{d\sigma_{\pi N}}{d\Omega} \xrightarrow[|t| \lesssim \text{const}]{s\to\infty} \frac{|b|^2 s}{(8\pi)^2} \sim \frac{|a|^2}{(8\pi)^2}. \tag{4.2}$$

Applying these same conditions to the amplitude for internal emission, gotten from Eq. (2.12) by ignoring the pole terms, we find the asymptotic high-energy behavior

$$|M_r|^2_{av}(\text{int}) \xrightarrow[s\to\infty]{} 8\pi^2 g^2(p-p')^2 m^{-4}\frac{d\sigma_{el}}{d\Omega}. \quad (4.3)$$

Here, as for Eq. (4.2), we have simply performed the spin-averaging trace and retained only the leading contributions in the limit that $s$ increases indefinitely while all relevant momentum transfers remain finite. Similarly, the asymptotic behavior when the soft pion is emitted from the chiral current of an external nucleon is

$$|M_r|^2_{av}(\text{ext}) \xrightarrow[s\to\infty]{} \frac{16g^2n^2(p-p')^2}{sm^2}\frac{d\sigma_{el}}{d\Omega}. \quad (4.4)$$

$M_r(\text{ext})$ comes from Eq. (2.12) by ignoring the anticommutator in the right-hand side of that equation. Isospin considerations in our formalism can be circumvented by considering production of neutral-soft pions.

We see that above some energy of the order of that of the nucleon rest mass the external-pion bremsstrahlung becomes relatively unimportant. It may also be, even before reaching energies which should be considered asymptotically high in the sense of our analysis above, that the dynamics of the interaction would show a phenomenological preference for processes in which the soft pion is not emitted from the chiral currents of the external nucleons. Among such processes are those described by the peripheral, or one-pion-exchange model for pion production. That model is supposed to pertain in the range of energies from the neighborhood of one BeV (lab KE) upward to around 4–5 BeV.

To whatever extent phenomenological analyses in terms of a one-pion-exchange model are successful or valid for pion-production processes, we can discount the nucleon-pole terms in our amplitude and examine the implications of the conservation of chirality for soft-pion emission at high energies in terms of only the $\gamma_5$ anticommutator part. It is, therefore, of interest to examine the relationship between our chirality conservation description and that of the peripheral model for these processes.

The amplitude (4.1) in the anticommutator part of formula (2.12) yields

$$iM_r = -(g/2m)\{\gamma_5, M\} = \gamma_5 ag/m. \quad (4.5)$$

In its elementary form the one-pion-exchange model gives for the corresponding amplitude for the same process,

$$iM_{\pi N\pi} = g\gamma_5 F(t)(\mu^2-t)^{-1}M_{\pi\pi}, \quad (4.6)$$

where $M_{\pi\pi}$ is the amplitude for pion-pion scattering, $\mu$ is the pion mass, and $F(t)$ is a function depending only on the invariant-nucleon-momentum transfer, $t$, and representing the combined effects of the $\gamma_5$ form factor as well as renormalization of the pion propagator and some of the off-the-mass-shell effects which modify the amplitude $M_{\pi\pi}$ for the internal scattering of the virtual pion.

The $\gamma_5$ anticommutator can act on the elastic amplitude so that in the sense of Feynman perturbation theory, a zero-energy external-meson line becomes attached to each internal-fermion propagator.[10] It might be imagined that in our treatment the $\gamma_5$ anticommutator operates in Eq. (4.5) upon the complete phenomenological elastic-scattering amplitude in such a manner that in the most peripheral, two-pion-exchange diagrams an external soft-pion line be attached to the nucleon propagator between the vertices of the two exchanged virtual pions. But also in standard radiation theory, emission and absorption of an external boson at a given vertex are operationally equivalent. Continuing in this vein, it might alternatively be imagined that the $\gamma_5$ vertex of one of these two exchanged pions become reabsorbed, liberating the corresponding pion-field operator to become the emitted-external pion at the peripheral interaction of the resulting one-pion-exchange production diagram. These two modes in which the $\gamma_5$ anticommutator could operate on the most peripheral contribution to the elastic-scattering amplitude are illustrated in Fig. 7.

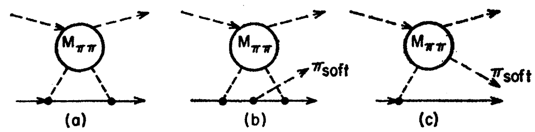


FIG. 7(a) Two-pion-exchange diagram, the most peripheral contribution to the phenomenological elastic scattering amplitude. (b) Two-pion-exchange diagram with a soft-pion line attached to the internal-nucleon propagator between $\gamma_5$ vertices of the exchanged pions. (c) One-pion-exchange diagram resulting from resorption of the $\gamma_5$ vertex of one of the internal pion lines and replacement with an externally emitted soft pion.

However, there are objections as to why the peripheral contributions to the chirality-conserving soft-pion emission described in this scheme could not account for any significant phenomenological cross section. First, the resorption of a $\gamma_5$ vertex in the two-pion-exchange elastic process would apply only to contributions from intermediate states of vanishing weight due to the restrictions on the soft-pion absorption (emission) operator inherent in our derivation. Second, it may be that the elastic scattering amplitude, $a$, occurring in Eq. (4.5) is largely imaginary in the energy domain of the peripheral model. (Diffraction peaking is well established in the BeV range,[19] although there are, to be sure, resonance effects observed there also.) On the other hand, it is a characteristic of the peripheral model that the proper energies of its internal virtual collisions are relatively low and the corresponding internal vertex amplitudes are, therefore, nearly real. To the extent that these were the cases, the chirality-conserving radiative amplitude, $M_r$ in Eq. (4.5), would be imaginary, while the corresponding amplitude of the peripheral model, $M_{\pi N\pi}$ in Eq. (4.6) would be real. So apparently, the peripheral model and our chirality-conservation formalism do not describe soft-pion production processes that overlap substantially.

If peripheral interactions are supposed to dominate all pion production in the 1–5 BeV energy region, then effects which violate the chiral symmetry are likely, in view of the preceding discussion, to be important. This is not very surprising when we recognize the central role played in peripheral processes by pion-pion interactions, and consider the likelihood that these are not chiral symmetric. For example, the familiar $s$-wave pion-pion interaction, $\lambda(\varphi_\alpha\varphi_\alpha)^2$, is not chiral invariant. Perhaps then we should treat the production of soft pions through peripheral interactions explicitly in terms of violations of the conservation of chirality.

A method for treating the effects of mild violations of chirality conservation in soft-pion production has been previously developed in NS. We will apply that method to the present problem to show that by treating the pion-pion interaction explicitly as a $\gamma_5$ symmetry-violating effect, the peripheral production of soft pions is indeed a result.

Let us assume that under an infinitesimal chiral transformation the change in the pion field results in a first-order infinitesimal change in the pion-pion interaction Hamiltonian:

$$\varphi \rightarrow \varphi+\delta\varphi\,, \quad H_\pi \rightarrow H_\pi+\delta H_\pi\,. \tag{4.7}$$

Then the soft-pion radiation formalism can be developed again as it was in Sec. II, except that the equation of motion for the total chirality, in addition to the terms which appeared in Eq. (2.2), now has another term that is due to the chirality's no longer commuting with the $S$ matrix:

$$\chi^{\text{out}}=S^{-1}\chi^{\text{in}}S=\chi^{\text{in}}+\int_{-\infty}^{\infty}\dot{\chi}dt\,. \tag{4.8}$$

Taking matrix elements of Eq. (4.7) between elastic states, exactly as in Sec. II, and ignoring this time the $[\chi_N,M]$ contribution, which we have already considered above; we get for the contribution from symmetry breaking effects in the pion-pion interaction,

$$\begin{aligned}\langle f^{\text{out}}|[S,\chi^{\text{in}}]|i^{\text{in}}\rangle &= \langle f^{\text{out}}|S\int_{-\infty}^{\infty}\dot{\chi}dt|i^{\text{in}}\rangle \\ &= i(2\pi)^4\delta^4(P_f+k(=0)-P_i) \\ &\qquad\times\langle f,k|M_r|i\rangle(2im/g)\,.\end{aligned} \tag{4.9}$$

The Heisenberg equation of motion for $\chi$ in terms of the Hamiltonian gives $i\dot{\chi}=[\chi,H]$, and our assumption is that only the pion-pion interaction contributes to this commutator; so $i\dot{\chi}=[\chi,H_\pi]$. But $\chi$ is the generator of the infinitesimal chiral transformations (4.7), and, consequently, the matrix element (4.9) can be written,

$$\langle f^{\text{out}}|Si\int_{-\infty}^{\infty}[H_\pi,\chi]dt|i^{\text{in}}\rangle = \langle f|\int(\delta H_\pi/\delta\varphi)d^4x|i\rangle\,. \tag{4.10}$$

Thus, our matrix element is just the radiative amplitude for emission associated with variation of the pion-pion interaction. In our case the variational field, $\delta\varphi$, represents the soft pion emitted from a (peripheral interaction) vertex involving three other pions, as depicted in Fig. 7(c).

The processes represented in the symmetry violating amplitude (4.9) might account for the discrepancies suggested in the comparison of the production cross-section magnitudes estimated on the one hand from the Shephard-Walker data at 1.3 BeV and those estimated on the other hand from our chirality-conservation calculation. If more thorough experimental data were to confirm that these pion-production processes are, in fact, dominated by symmetry-violating peripheral interactions, then the angular distributions calculated without taking them into account would contain very little physical relevance. It may turn out that symmetry-conserving and symmetry-violating effects *both* contribute; the former processes being, in general, more important at large-momentum transfers to the nucleon, and the peripheral interactions being more important at small-momentum transfers.

Applying Eq. (3.8) to the cross section calculated with the amplitude (4.6), we can estimate the contribution of the peripheral processes in the strip of phase space corresponding to the least inelastic events. The magnitudes calculated in this way using only the $S$-wave pion-pion interaction are about 1–2 times as large as those estimated in Sec. III for the chirality-conserving contributions. Using the total phenomenological pion-pion interaction, the peripheral processes contribute a $\sigma_\epsilon$ that is somewhat more than half the experimental estimate. To make these estimations we assume that the peripheral model can be cogently applied even at large momentum transfers. For this purpose the prescription of Ferrari and Selleri for continuing the pseudoscalar form factor and pion propagator away from the pion pole were directly adapted.[20]

The questions of the relative importance of chirality conserving and nonconserving effects in these processes and the agreement or disagreement of either with experiment can be resolved when more experimental information is accumulated. In view of these ambiguities which confront us in the present problem, it seems to be advantageous to try to investigate chirality conservation with phenomena in which symmetry-breaking effects are absent (if there are any), or else where they dominate. The production of soft pions by electromagnetic and weak interactions, as treated in NS, are promising examples of the latter category; and, in particular, electroproduction data will likely yield important information on several aspects of chirality conservation and its violation. If our preliminary estimates of the relative unimportance of $\gamma_5$ symmetry-conserving contributions to the process $\pi N\rightarrow\pi N\pi_{\text{soft}}$ are confirmed with conclusive data, then these processes will fall into the same category.

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[19] See, for instance Figs. 3 and 4 of Ref. 18.

[20] E. Ferrari and F. Selleri, Suppl. Nuovo Cimento **24**, 389 (1962); Phys. Rev. Letters **7**, 387 (1961).